Middle Years Programme

Mathematics guide (for use from September 2020/January 2021)

International Baccalaureate®
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The International Baccalaureate Organization (known as the IB) offers four high-quality and challenging educational programmes for a worldwide community of schools, aiming to create a better, more peaceful world. This publication is one of a range of materials produced to support these programmes.

IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

As IB learners we strive to be:

INQUIRERS
We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

KNOWLEDGEABLE
We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

THINKERS
We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

COMMUNICATORS
We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

PRINCIPLED
We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

OPEN-MINDED
We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

CARING
We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

RISK-TAKERS
We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

BALANCED
We understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

REFLECTIVE
We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.

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Purpose of this guide

This guide is for use from September 2020 or January 2021, depending on the start of the school year.

This document provides the framework for teaching and learning in mathematics in the Middle Years Programme (MYP) and must be read and used in conjunction with the document *MYP: From principles into practice*, which includes:

- general information about the programme
- the MYP unit planner, with guidance for developing the curriculum that is relevant for all subject groups
- detailed information about approaches to learning
- advice that supports access and inclusion (including accommodations for students with learning support requirements)
- a statement on academic integrity.

In MYP publications, requirements appear in a text box like this one.

Additional resources

Teacher support materials (TSMs) are available in the programme resource centre (resources.ibo.org). The TSM for mathematics contains support for developing the written, taught and assessed curriculum. It provides examples of good practice, including subject-group overviews, assessment tasks and markschemes, as well as student work with teacher comments.

An optional process of external assessment can lead to **IB MYP course results** for mathematics courses, and these results can contribute to the awarding of an **IB MYP certificate**. More information is available in the annual publication Middle Years Programme *Assessment procedures*.

A range of publications that support the MYP are available at the IB store (store.ibo.org).

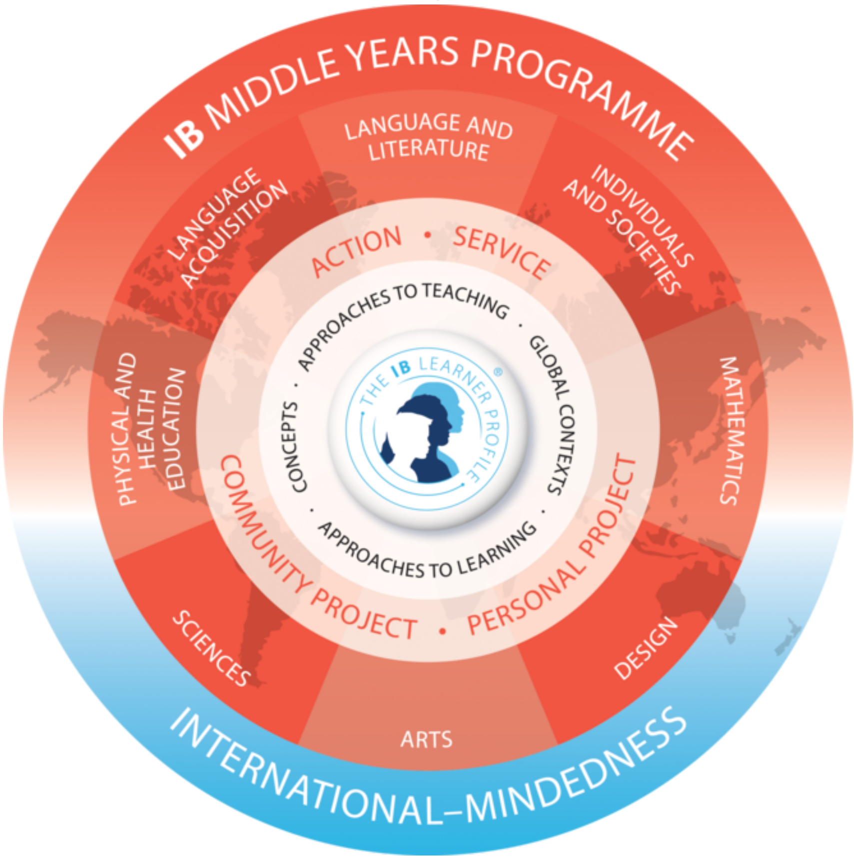
Acknowledgments

The IB gratefully acknowledges the generous contributions of IB World Schools and a global community of educators who collaborate in the development of the MYP.

Programme model

Figure 1

Middle Years Programme model



The MYP is designed for students aged 11 to 16. It provides a framework of learning that encourages students to become creative, critical and reflective thinkers. The MYP emphasizes intellectual challenge, encouraging students to make connections between their studies in traditional subjects and the real world. It fosters the development of skills for communication, intercultural understanding and global engagement —essential qualities for young people who are becoming global leaders.

The MYP is flexible enough to accommodate the demands of most national or local curriculums. It builds upon the knowledge, skills and attitudes developed in the IB Primary Years Programme (PYP) and prepares students to meet the academic challenges of the IB Diploma Programme (DP) and the IB Career-related Programme (CP).

The MYP:

- addresses holistically students' intellectual, social, emotional and physical **well-being**
- provides students opportunities to develop the **knowledge, attitudes and skills** they need in order to manage complexity, and take responsible action for the future
- ensures breadth and depth of understanding through study in **eight subject groups**

- requires the study of at least **two languages** to support students in understanding their own cultures and those of others
- empowers students to participate in **service with the community**
- helps to prepare students for **further education**, the **workplace** and a **lifetime of learning**.

Nature of mathematics

> Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world.
>
> Roger Bacon (1214–1294)

The study of mathematics is a fundamental part of a balanced education. It promotes a powerful universal language, analytical reasoning and problem-solving skills that contribute to the development of logical, abstract and critical thinking. Mathematics can help make sense of the world and allows phenomena to be described in precise terms. It also promotes careful analysis and the search for patterns and relationships, skills necessary for success both inside and outside the classroom. Mathematics, then, should be accessible to, and studied by, all students.

Studying mathematics, however, should be more than simply learning formulae or rules. Students should not have the impression that all of the answers to mathematics can be found in a book but, rather, that they can be active participants in the search for concepts and relationships. In that light, mathematics becomes a subject that is alive with the thrill of exploration and the rewards of discovery. At the same time, that new knowledge may then be applied to other situations, opening up even more doors for students. MYP mathematics promotes both inquiry and application, helping students to develop problem-solving techniques that transcend the discipline and that are useful in the world outside school.

An MYP mathematics programme should be tailored to the needs of students, seeking to intrigue and motivate them to want to learn its principles. Students should see authentic examples of how mathematics is useful and relevant to their lives and be encouraged to apply it to new situations. Mathematics provides the foundation for the study of sciences, engineering and technology. However, it is also evident in the arts and is increasingly important in economics, the social sciences and the structure of language. Students in the MYP are encouraged to use information and communication technology (ICT) tools to represent information, to explore and model situations, and to find solutions to various problems. These are skills that are useful in a wide range of arenas. MYP mathematics aims to equip all students with the knowledge, understanding and intellectual capabilities to address further courses in mathematics, as well as to prepare those students who will use mathematics in their studies, workplaces and lives in general.

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Mathematics across the IB continuum

The IB continuum of international education provides a progression of learning for students aged 3–19. In the PYP, mathematics is viewed primarily as a vehicle to support inquiry, providing a universal language through which we make sense of the world around us. It is intended that students become competent users of the language of mathematics and begin to use it as a way of thinking, as opposed to seeing mathematics as a series of facts and equations to be memorized. It is also recognized that students can appreciate the intrinsic fascination of mathematics and explore the world through its unique perceptions. In the same way that students describe themselves as "authors" or "artists", a school's mathematics programme should also provide students with the opportunity to see themselves as "mathematicians", who enjoy and are enthusiastic about exploring and learning about the subject. MYP mathematics aims to build on what students learn and do in the PYP and other student-centred programmes of primary education. There are no prior formal learning requirements; however, a list of suggested prior learning is provided with the mathematics framework in this guide.

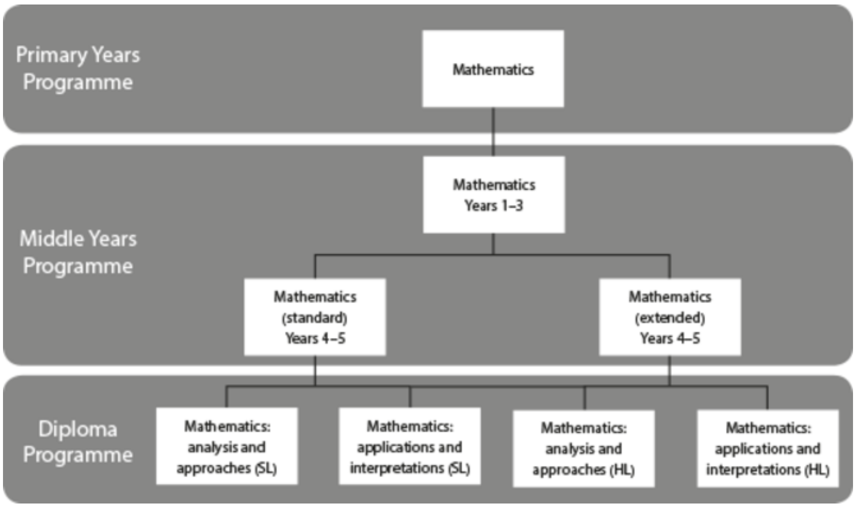
In both the PYP and the MYP, it is important that learners acquire mathematical understanding by constructing their own meaning through increasing levels of abstraction, starting with an exploration of their own personal experiences, understandings and knowledge. Additionally, it is fundamental to the philosophy of both programmes that, since it is to be used in real-life situations, mathematics needs to be taught in relevant, realistic contexts, rather than by attempting to impart a fixed body of knowledge. In both programmes, mathematics is valued not only for its beauty but also for its usefulness in helping us to understand how the world works and for providing us with a unique way to communicate. Mathematics is an essential tool for transdisciplinary and interdisciplinary inquiry. Teaching and learning experiences in both the PYP and MYP challenge students to be curious, ask questions and explore and interact with the environment physically, socially and intellectually. Through engaging in this process, students are able to construct meaning about mathematics concepts, transfer this meaning to symbols and apply mathematical understanding in familiar and unfamiliar situations.

MYP mathematics courses help specifically to prepare students for the study of DP mathematics courses. As students progress from the MYP to the DP or CP, the emphasis on understanding increases as students work towards developing a strong mathematical knowledge base that will allow them to study a wide range of topics. Through this process they also work on communicating their ideas in ways that allow others to understand their thinking. The MYP mathematics objectives and criteria have been developed with both the internal and external assessment requirements of the DP in mind. The use of technology, which is required in DP mathematics, is also emphasized in the MYP as a tool for learning, applying and communicating mathematics. Where students in the MYP may select either standard or extended mathematics, the DP mathematics curriculum offers two courses, each of which is available at standard level (SL) and higher level (HL)—mathematics: analysis and approaches, and mathematics: applications and interpretation. MYP students enrolled in extended mathematics generally elect to take one of the DP mathematics courses at HL. Students in MYP standard mathematics should seek the recommendation of their teacher when deciding which course to pursue in the DP, as HL mathematics is rarely an appropriate course of study for these students.

Figure 2 shows the IB continuum pathways to DP courses in mathematics.

Figure 2

Pathway to DP courses—mathematics: analysis and approaches (SL), mathematics: applications and interpretations (SL), mathematics: analysis and approaches (HL), and mathematics: applications and interpretations (HL)



Regardless of the options available to students, possible general strategies teachers can use in developing a smooth transition between MYP and DP mathematics courses include:

- facilitating mathematics vertical planning sessions between the MYP and the DP
- developing an understanding and consistent use of a common set of key terms, notation and formulae that are applicable to all programmes
- preparing students to develop effective strategies for external examinations as well as inquiry-based learning across all mathematics courses
- providing students with the opportunity to explore problems that incorporate several areas of mathematics
- providing students with the opportunity to solve problems using mathematical concepts in unfamiliar situations.

The knowledge, skills and attitudes that students develop in mathematics courses provide a meaningful foundation for further study and help to prepare students for careers in, for example, climate research, actuary and insurance work, public-policy development, engineering, financial analysis and economic development, research and analysis, software development, biostatistics and epidemiology, law or medicine.

Aims

The aims of all MYP subjects state what a teacher may expect to teach and what a student may expect to experience and learn. These aims suggest how the student may be changed by the learning experience.

The aims of MYP mathematics are to encourage and enable students to:

- enjoy mathematics, develop curiosity and begin to appreciate its elegance and power
- develop an understanding of the principles and nature of mathematics
- communicate clearly and confidently in a variety of contexts
- develop logical, critical and creative thinking
- develop confidence, perseverance, and independence in mathematical thinking and problem-solving
- develop powers of generalization and abstraction
- apply and transfer skills to a wide range of real-life situations, other areas of knowledge and future developments
- appreciate how developments in technology and mathematics have influenced each other
- appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
- appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
- appreciate the contribution of mathematics to other areas of knowledge
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- develop the ability to reflect critically upon their own work and the work of others.

Objectives

The objectives of any MYP subject group state the specific targets that are set for learning in the subject. They define what the student will be able to accomplish as a result of studying the subject.

The objectives of MYP mathematics encompass the factual, conceptual, procedural and metacognitive dimensions of knowledge.

Schools **must** use the objectives provided in this guide for years 1, 3 and 5 of the programme.

Each objective is elaborated by a number of **strands**; a strand is an aspect or indicator of the learning expectation.

Subject groups **must** address **all** strands of **all** four objectives **at least twice** in **each year** of the MYP.

These objectives relate directly to the assessment criteria found in the "Assessed curriculum" section of this guide.

A Knowing and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop skills. This objective assesses the extent to which students can select and apply mathematics to solve problems in both familiar and unfamiliar situations in a variety of contexts.

This objective requires students to demonstrate knowledge and understanding of the concepts and skills of the four branches in the prescribed framework (numerical and abstract reasoning, thinking with models, spatial reasoning, and reasoning with data).

In order to reach the aims of mathematics, students should be able to:

i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. apply the selected mathematics successfully when solving problems

iii. solve problems correctly in a variety of contexts.

B Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Working through investigations encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 6 (for years 1 and 2) and a maximum achievement level of 4 (for year 3 and up). However, teachers should give enough direction to ensure that all students can begin the investigation.

For year 3 and up, a student who describes a general rule consistent with incorrect findings will be able to achieve a maximum achievement level of 6, provided that the rule is of an equivalent level of complexity.

In order to reach the aims of mathematics, students should be able to:

i. select and apply mathematical problem-solving techniques to discover complex patterns

ii. describe patterns as general rules consistent with findings

iii. prove, or verify and justify, general rules.

C Communicating

Mathematics provides a powerful and universal language. Students are expected to use appropriate mathematical language and different forms of representation when communicating mathematical ideas, reasoning and findings, both orally and in writing.

In order to reach the aims of mathematics, students should be able to:

i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations
ii. use appropriate forms of mathematical representation to present information
iii. move between different forms of mathematical representation
iv. communicate complete, coherent and concise mathematical lines of reasoning
v. organize information using a logical structure.

D Applying mathematics in real-life contexts

MYP mathematics encourages students to see mathematics as a tool for solving problems in an authentic real-life context. Students are expected to transfer theoretical mathematical knowledge into real-world situations and apply appropriate problem-solving strategies, draw valid conclusions and reflect upon their results.

In order to reach the aims of mathematics, students should be able to:

i. identify relevant elements of authentic real-life situations
ii. select appropriate mathematical strategies when solving authentic real-life situations
iii. apply the selected mathematical strategies successfully to reach a solution
iv. justify the degree of accuracy of a solution
v. justify whether a solution makes sense in the context of the authentic real-life situation.

Planning a progression of learning

MYP mathematics relies on a progression in the complexity of the level of mathematics throughout the programme. For this reason, the objectives listed below for years 1, 3 and 5 are quite similar; however, the complexity of the mathematics being assessed is increasing. Throughout the programme, students should engage with the curriculum and demonstrate their understanding at increasing levels of sophistication.

Year 1 In order to reach the aims of mathematics, students should be able to:	Year 3 In order to reach the aims of mathematics, students should be able to:	Year 5 In order to reach the aims of mathematics, students should be able to:
Objective A: Knowing and understanding		
i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations ii. apply the selected mathematics successfully when solving problems iii. solve problems correctly in a variety of contexts.	i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations ii. apply the selected mathematics successfully when solving problems iii. solve problems correctly in a variety of contexts.	i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations ii. apply the selected mathematics successfully when solving problems iii. solve problems correctly in a variety of contexts.
Objective B: Investigating patterns		
i. apply mathematical problem-solving techniques to recognize patterns ii. describe patterns as relationships or general rules consistent with correct findings iii. verify whether the pattern works for other examples.	i. select and apply mathematical problem-solving techniques to discover complex patterns ii. describe patterns as relationships and/or general rules consistent with findings iii. verify and justify relationships and/or general rules.	i. select and apply mathematical problem-solving techniques to discover complex patterns ii. describe patterns as general rules consistent with findings iii. prove, or verify and justify, general rules.
Objective C: Communicating		
i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written statements ii. use appropriate forms of mathematical representation to present information iii. communicate coherent mathematical lines of reasoning	i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations ii. use appropriate forms of mathematical representation to present information iii. move between different forms of mathematical representation	i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations ii. use appropriate forms of mathematical representation to present information iii. move between different forms of mathematical representation

Year 1 In order to reach the aims of mathematics, students should be able to:	Year 3 In order to reach the aims of mathematics, students should be able to:	Year 5 In order to reach the aims of mathematics, students should be able to:
iv. organize information using a logical structure.	iv. communicate complete and coherent mathematical lines of reasoning v. organize information using a logical structure.	iv. communicate complete, coherent and concise mathematical lines of reasoning v. organize information using a logical structure.
Objective D: Applying mathematics in real-life contexts		
i. identify relevant elements of authentic real-life situations ii. select appropriate mathematical strategies when solving authentic real-life situations iii. apply the selected mathematical strategies successfully to reach a solution iv. explain the degree of accuracy of a solution v. describe whether a solution makes sense in the context of the authentic real-life situation.	i. identify relevant elements of authentic real-life situations ii. select appropriate mathematical strategies when solving authentic real-life situations iii. apply the selected mathematical strategies successfully to reach a solution iv. explain the degree of accuracy of a solution v. explain whether a solution makes sense in the context of the authentic real-life situation.	i. identify relevant elements of authentic real-life situations ii. select appropriate mathematical strategies when solving authentic real-life situations iii. apply the selected mathematical strategies successfully to reach a solution iv. justify the degree of accuracy of a solution v. justify whether a solution makes sense in the context of the authentic real-life situation.

The range of assessed skills, techniques and strategies as well as the complexity of their application, must increase as students progress through the programme.

Interdisciplinary learning

Interdisciplinary teaching and learning is grounded in individual subject groups and disciplines, but extends disciplinary understanding in ways that are:

- **integrative**—bringing together concepts, methods, or modes of communication from two or more subject groups, disciplines, or established areas of expertise to develop new perspectives
- **purposeful**—connecting disciplines to solve real-world problems, create products or address complex issues in ways that would have been unlikely through a single approach.

Interdisciplinary teaching and learning builds a connected curriculum that addresses the developmental needs of students in the MYP. It prepares students for further academic (inter)disciplinary study and for life in an increasingly interconnected world.

The MYP uses concepts and contexts as starting points for meaningful integration and transfer of knowledge across subject groups and disciplines. *Fostering interdisciplinary teaching and learning in the MYP* (August 2014, updated September 2017) contains more information, including a detailed process for planning and recording interdisciplinary units.

MYP schools are responsible for engaging students in at least one collaboratively planned interdisciplinary unit for each year of the programme.

MYP mathematics offers many opportunities for interdisciplinary teaching and learning. Possible interdisciplinary units in this subject group could include inquiries into:

- collecting and analysing statistical data in physical and health education classes
- applying geometry knowledge in design projects
- investigating the links between musical theory and mathematical sequences.

Interdisciplinary learning can take place through both large- and small-scale learning engagements. Authentic interdisciplinary learning often requires critical reflection and detailed collaborative planning. However, teachers and students can also make interdisciplinary connections through spontaneous learning experiences and conversations.

All MYP subject-group teachers are responsible for developing meaningful ongoing opportunities for interdisciplinary teaching and learning.

MYP projects

The MYP community project (for students in years 3 or 4) and MYP personal project (for students in year 5) aim to encourage and enable sustained inquiry within a global context that generates new insights and deeper understanding. In these culminating experiences, students develop confidence as principled, lifelong learners. They grow in their ability to consider their own learning, communicate effectively and take pride in their accomplishments.

Courses in mathematics help students to develop key approaches to learning (ATL) that lead to success and enjoyment in the MYP projects. In this subject group, students have important opportunities to practise ATL skills, especially cognitive skills. Organizing and transforming information are essential aspects of mathematics.

From their learning experiences in this subject group, students can find inspiration for their projects. Through the application of mathematics in real-life situations, students will be able to see a multitude of opportunities to incorporate their mathematical skills into the projects.

Mathematics offers many opportunities for learning through action. Inspiration from mathematics for community projects and personal projects might include inquiries into:

- the statistical analysis of a local or global sustainability issue
- the mathematical analysis of athletic performance by a team or individual
- developing networking solutions for transport routes to and from school.

For further information please refer to the *Projects guide* (May 2014, updated March 2018), which can be found in the programme resource centre under **MYP resources**>**Curriculum**>**Core elements**>**MYP Projects**.

Requirements

Teaching hours

Schools must allocate the teaching hours necessary to meet the requirements of MYP mathematics.

The MYP requires at least 50 hours of teaching time for each subject group in each year of the programme.

In practice, more time is often necessary to meet subject-group aims and objectives and to provide for the sustained, concurrent teaching that enables interdisciplinary study.

For students pursuing IB MYP course results that can contribute to the awarding of the IB MYP certificate, mathematics courses should include at least 70 teaching hours in each of the final two years of the programme (MYP year 4 and MYP year 5).

Organizing mathematics in the school

All MYP subjects, including mathematics, provide a curricular framework with set final aims and objectives. MYP mathematics also provides a framework of topics and skills organized into four branches at three different levels: MYP years 1–3, MYP years 4–5 at a standard level, and MYP years 4–5 at an extended level. The framework also includes suggestions for enrichment topics; these are optional and are not included in eAssessments.

Levels of mathematics in MYP years 4 and 5

MYP mathematics should be accessible to, and studied by, all students. Schools must ensure that the mathematics curriculum allows all students the opportunity to reach their full potential and achieve the final aims and objectives of MYP mathematics. The framework for mathematics is organized so that students in years 4 and 5 can work at two levels of challenge: **standard mathematics** and **extended mathematics**.

- **Standard mathematics** aims to give all students a sound knowledge of mathematical principles while allowing them to develop the skills needed to meet the objectives of MYP mathematics.
- **Extended mathematics** consists of the standard mathematics framework supplemented by additional topics and skills. This level provides the foundation for students who wish to pursue further studies in mathematics: for example, HL mathematics courses as part of the DP. Extended mathematics provides greater breadth and depth to the standard mathematics framework.

Within the prescribed framework, all MYP mathematics courses should ensure that students:

- apply mathematics to authentic real-life situations
- perform investigations to discover patterns.

Planning the mathematics curriculum

IB World Schools are responsible for developing and structuring MYP mathematics courses that provide opportunities for students to meet the aims and objectives of the programme. Each school's circumstances, including local and national curriculum requirements, determine the organization of mathematics within the school.

MYP standards and practices require schools to facilitate and promote collaborative planning for the purpose of curriculum development and review.

Mathematics objectives for years 1–5 of the curriculum provide continuity and outline a progression of learning. These objectives guide teachers in making decisions about developmentally appropriate learning experiences, including formative and summative assessments.

As they develop the vertical articulation of mathematics over the years of the programme, teachers should plan increasingly complex units of work that encompass multiple objectives. However, within these units, discrete tasks or smaller units of work might concentrate on specific objectives or individual strands.

Mathematics courses offer many opportunities to build interdisciplinary connections across the curriculum. Horizontal articulation for each year of the programme should coordinate teaching and learning across courses in mathematics, as well as identify shared conceptual understandings and ATL that span multiple subject groups and help to create a coherent learning experience for students throughout the year.

In mathematics, learning is generally sequential. Success in later mathematics courses relies on building fundamentals in earlier ones. Not only content but also teaching methods, assessments and problem-solving strategies need to be sequenced appropriately. Teachers are encouraged to articulate the mathematics continuum in their schools. This may be done in a number of ways; however, a planned approach is necessary. Consideration of knowledge, skills and attitudes is required to align learning and assessment vertically. The following examples demonstrate possible ways that activities can be sequenced to prepare students for future DP mathematics content. These examples demonstrate different approaches teachers could take when planning a smooth transition from MYP year 1 through to mathematics in the DP.

Example 1: Modelling anthropometric (body-part) measurements

This example demonstrates a method that could be used to sequence skills required for an internal assessment. This example focuses on activities to model anthropometric measurements (measurement of the human individual) from MYP year 1 to the DP.

Year	Topics/skills	Possible activities
MYP 1	• Data collection • Graphical representations • Relating variables • Informal and formal description of observed trends	Simple models, such as height of person versus length of foot
MYP 3	• Cartesian plane • Plotting points • Trends and predictions • Modelling "by eye"	Modelling data: for example, male versus female

Year	Topics/skills	Possible activities
	• Independent versus dependent variables • Appropriate values for variables	
MYP 5	• Domain and range • Linear regression • Accuracy (correlation) • Prediction • Use of technology	Modelling-based extensions of the domain and range: for example, height (male or female) versus length of foot (male or female) Solving crimes: If you have the length of the foot, how tall is the person likely to be? Can giants exist?
DP 1–2	• Choosing own models • Comparing • Correlation coefficient • Use of more complex models • Students discuss limitations of their models and data	Modelling based on a variety of functions Application of paleo-anthropometry (relating sizes of bones to determine the height of a dinosaur/pre-human)

Example 2: Volume and area

This example demonstrates a method that could be used for sequencing a specific problem. This example focuses on activities to investigate volume and area from MYP year 1 to the DP.

Year	Possible activities
MYP 1	Students will investigate approximation by using a cylinder to approximate the volume for a variety of irregularly shaped beverage containers. They will produce a report to describe and explain which containers are best approximated by a cylinder.
MYP 3	Students will investigate the effects of changing the dimensions (radius and height) of a cylindrical container and produce a report describing which dimension has the greatest effect on surface area and volume.
MYP 5	Students will determine the dimensions of a can of soft drink that will minimize its surface area given a fixed volume (330 ml) through the use of graphing (using a graphic display calculator [GDC]).
DP 1–2	Students will determine the dimensions of a can of soft drink that will minimize its surface area given a fixed volume (330 ml) through the use of calculus and compare it to the value produced using another method.

Work that can be done to ease the transition from MYP to DP in any of these areas will help students be more successful and the previous examples need not be construed as the only means of accomplishing this.

Teaching and learning through inquiry

Inquiry, in the broadest sense, is the process that is used to move to deeper levels of understanding. Inquiry involves speculating, exploring, questioning and connecting. In all IB programmes, inquiry develops curiosity and promotes critical and creative thinking.

The MYP structures sustained inquiry in mathematics by developing **conceptual understanding** in **global contexts**. Teachers and students develop a **statement of inquiry** and use **inquiry questions** to explore the subject. Through their inquiry, students develop specific interdisciplinary and disciplinary **approaches to learning skills**.

Conceptual understanding

A concept is a "big idea"—a principle or notion that is enduring, the significance of which goes beyond particular origins, subject matter or place in time. Concepts represent the vehicle for students' inquiry into the issues and ideas of personal, local and global significance, providing the means by which they can explore the essence of mathematics.

Concepts have an important place in the structure of knowledge that requires students and teachers to think with increasing complexity as they organize and relate facts and topics.

Concepts express understanding that students take with them into lifelong adventures of learning. They help students to develop principles, generalizations and theories. Students use conceptual understanding as they solve problems, analyse issues and evaluate decisions that can have an impact on themselves, their communities and the wider world.

In the MYP, conceptual understanding is framed by prescribed key and related concepts. Teachers must use these concepts to develop the curriculum. Schools may identify and develop additional concepts to meet local circumstances and curriculum requirements.

Key concepts

Key concepts promote the development of a broad curriculum. They represent big ideas that are both relevant within and across disciplines and subjects. Inquiry into key concepts can facilitate connections between and among:

- courses within the mathematics subject group (intra-disciplinary learning)
- other subject groups (interdisciplinary learning).

Table 1 lists the key concepts to be explored across the MYP. The key concepts contributed by the study of mathematics are **form**, **logic** and **relationships**.

Table 1

MYP key concepts

Aesthetics	Change	Communication	Communities
Connections	Creativity	Culture	Development
Form	Global interactions	Identity	**Logic**
Perspective	**Relationships**	Systems	Time, place and space

These key concepts inform units of work and help to organize teaching and learning.

Form

Form is the shape and underlying structure of an entity or piece of work, including its organization, essential nature and external appearance.

Form in MYP mathematics refers to the understanding that the underlying structure and shape of an entity is distinguished by its properties. Form provides opportunities for students to appreciate the aesthetic nature of the constructs used in a discipline.

Logic

Logic is a method of reasoning and a system of principles used to build arguments and reach conclusions.

Logic in MYP mathematics is used as a process in making decisions about numbers, shapes, and variables. This system of reasoning provides students with a method for explaining the validity of their conclusions. Within the MYP, this should not be confused with the subfield of mathematics called “symbolic logic”.

Relationships

Relationships are the connections and associations between properties, objects, people and ideas—including the human community’s connections with the world in which we live. Any change in relationship brings consequences—some of which may occur on a small scale, while others may be far reaching, affecting large networks and systems such as human societies and the planetary ecosystem.

Relationships in MYP mathematics refers to the connections between quantities, properties or concepts and these connections may be expressed as models, rules or statements. Relationships provide opportunities for students to explore patterns in the world around them. Connections between the student and mathematics in the real world are important in developing deeper understanding.

Other key concepts can also be important in mathematics and these are outlined in the “**Mathematics framework” chapter** of this guide.

Related concepts

Related concepts promote deep learning. They are grounded in specific disciplines and are useful for exploring key concepts in greater detail. Inquiry into related concepts helps students develop more complex and sophisticated conceptual understanding. Related concepts may arise from the subject matter of a unit or from the craft of a subject—that is, its features and processes.

Table 2 lists related concepts for the study of mathematics. Teachers are not limited to the related concepts listed in this chart and may choose others when planning units, including from other subject groups.

Table 2

Related concepts in mathematics

Approximation	Change	Equivalence
Generalization	Models	Patterns
Quantity	Representation	Simplification
Space	Systems	Validity

The appendices contain a glossary of these related concepts for mathematics.

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Global contexts for teaching and learning

Global contexts direct learning towards independent and shared inquiry into our common humanity and shared guardianship of the planet. Using the world as the broadest context for learning, MYP mathematics can develop meaningful explorations of:

- identities and relationships

- orientation in space and time
- personal and cultural expression
- scientific and technical innovation
- globalization and sustainability
- fairness and development.

Teachers must identify a global context for teaching and learning, or develop additional contexts that help students explore the relevance of their inquiry (why it matters).

Many inquiries into mathematics concepts naturally focus on scientific and technical innovation. However, courses in this subject group should, over time, offer students multiple opportunities to explore all MYP global contexts in relation to the aims and objectives of the subject group.

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Statements of inquiry

Statements of inquiry set conceptual understanding in a global context in order to frame classroom inquiry and direct purposeful learning. Table 3 shows some possible statements of inquiry for MYP mathematics units.

Table 3

Example statements of inquiry

Statement of inquiry	Key concept Related concepts Global context	Possible project/study
The design of structures is often influenced by the responsible use of finite resources.	• Form • Space • Quantity • Fairness and development	Geometry and trigonometry—volume
Logic is a powerful tool for evaluating the validity of what we discover through measurement and observation.	• Logic • Validity • Orientation in space and time	Geometry and trigonometry—parallel lines and transversals
Decision-making can be improved by using a model to represent relationships.	• Relationships • Models • Representation • Identities and relationships	Algebra—quadratic functions
Understanding patterns in forms and space can enhance creativity.	• Form • Patterns • Space • Personal and cultural expression	Geometry and trigonometry—transformations
Modelling using a logical process that involves the simplification of patterns helps us to understand the world.	• Logic • Patterns • Simplification • Models	Algebra—projectile motion

Statement of inquiry	Key concept Related concepts Global context	Possible project/study
	• Scientific and technical innovation	
Relationships and the changes that affect them can help us understand human impact on environmental systems.	• Relationships • Systems • Change • Globalization and sustainability	Number—exponentials and logarithms
Principles and discoveries often arise when patterns in the natural world are described as relationships.	• Relationships • Patterns • Scientific and technical innovation	Statistics and probability—line of best fit

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Inquiry questions

Teachers and students use statements of inquiry to help them identify factual, conceptual and debatable inquiry questions. Inquiry questions give direction to teaching and learning, and they help to organize and sequence learning experiences.

Table 4 shows some possible inquiry questions for MYP mathematics units.

Table 4

Examples of factual, conceptual and debatable questions

Factual questions: Remembering facts and topics	Conceptual questions: Analysing big ideas	Debatable questions: Evaluating perspectives and developing theories
• How do the gradients of perpendicular lines compare? • How does the volume of a quantity differ from its area? • What determines whether two events are independent?	• What does it mean to have a “solution” of a function? • Why can approximation be useful? • How could we map the neural network of a human brain?	• What is more natural: order or chaos? • Are all events in the universe determined by probability? • How big is infinity?

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Approaches to learning

All MYP units of work offer opportunities for students to develop and practise ATL skills. These skills provide valuable support for students working to meet the aims and objectives of the subject group.

The ATL skills are grouped into five categories that span the IB continuum of international education, and IB programmes identify discrete skills in each category that can be introduced, practised and consolidated in the classroom and beyond.

While ATL skills are relevant across all MYP subject groups, teachers may also identify ATL skill indicators especially relevant for, or unique to, a particular subject group or course.

Table 5 suggests some of the indicators that can be important in mathematics.

Table 5

Examples of mathematics-specific skill indicators

Category	Skill indicator
Thinking skills	Use prioritization and order of precedence in problem-solving.
Social skills	Help others to create success for themselves during group work.
Communication skills	Organize and interpret data using both analogue and digital tools.
Self-management skills	Practise focus and concentration while solving multiple problems.
Research skills	Use a variety of technologies and media platforms, including social media and online networks, to source information.

Well-designed learning engagements and assessments provide rich opportunities for students to practise and demonstrate ATL skills. Each MYP unit explicitly identifies ATL skills around which teaching and learning can focus, and through which students can authentically demonstrate what they are able to do. Formative assessments provide important feedback for developing discrete skills, and many ATL skills support students as they demonstrate their achievements in summative assessments of subject-group objectives.

Table 6 lists some specific ATL skills that students can demonstrate through performances of understanding in mathematics.

Table 6

Examples of demonstrations of ATL skills in mathematics

Approaches to learning
Thinking (critical thinking): draw justifiable conclusions and generalizations from investigating patterns. **Communication (reflection):** keep a regular journal during the investigation to maintain a record of reflections.

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Mathematics framework

The framework for MYP mathematics outlines four branches of mathematical study.

- Numerical and abstract reasoning
- Thinking with models
- Spatial reasoning
- Reasoning with data

Schools can use the framework for mathematics as a tool for curriculum mapping when designing and planning their mathematics courses. **Schools are not expected to address all the branches of the framework in each year of the programme, nor are they required to teach every topic or skill suggested in the framework.** However, over the five years (or complete duration) of the programme, students should experience learning in all four branches of the framework for mathematics.

The topics and skills are **examples** of what students may expect to study in years 1–3 of the programme, and at the two levels—standard mathematics and extended mathematics—in years 4 and 5 of the programme. The framework also includes suggestions for enrichment topics; these are optional and are not included in eAssessments. A skill listed as **extended** or **enrichment** in the framework could be addressed in a standard mathematics class in some cases. Schools are responsible for defining the distinction between standard and extended mathematics courses, and for determining when it is appropriate to include enrichment topics and skills in units of inquiry.

Recommended prior learning

The MYP mathematics framework is based on the expectation that students in MYP year 1 have at least five years of prior mathematics experience and are familiar with the following topics and skills. Teachers must therefore ensure that any skills listed here that are unknown to their students at the start of the course are included at an early stage. Teachers should also take into account the existing mathematical knowledge of their students in designing an appropriate course of study for mathematics.

In order to work towards the MYP mathematics objectives and develop the skills in the mathematics framework, students should be able to:

- use the base 10 place value system to represent the magnitude of very large and very small quantities
- use the associative and commutative properties to express quantities and expressions in multiple equivalent forms
- use the relationships between the operations of addition, subtraction, multiplication and division in order to process information to solve problems
- use fractions and decimals to represent whole-part relationships
- model complex operations in a variety of ways, such as algorithms or flow charts
- analyse and identify rules for patterns
- use functions or rules to extend patterns and describe patterns
- apply a range of procedures to measure different attributes of objects and events
- describe the accuracy of measurements with reference to the situation and the precision of the tool
- convert units and measurements to describe the world we live in in terms that make sense
- transform shapes using rotation, reflection and translation
- use geometric vocabulary to show that changing the position of a shape does not alter its properties

- use geometric shapes and vocabulary useful to represent and describe objects and events in real-world situations
- collect, organize, display and interpret data in different ways
- use different graph forms to highlight different aspects of data
- express probability in numerical notations
- use data to describe the probability of events in daily life.

Numerical and abstract reasoning

The ability to work with numbers and symbols is an essential skill in mathematics. Students are expected to have an understanding of number concepts and to develop the skills of calculation and approximation. Algebra uses letters and symbols to represent numbers, quantities and operations, and employs variables to solve mathematical problems. Algebra is an abstraction of the concepts first used when dealing with numbers and is essential for further learning in mathematics. Students should understand that the use of numbers and symbols to express patterns and to describe real-life situations goes back to humankind's earliest beginnings, and that mathematics has multicultural roots.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the numerical and abstract reasoning branch include **change** (ratios, number bases), **communication** (number lines, units of measurement), **connections** (number bases, number sequences, Venn diagrams), **development** (number sequences, prime numbers), **identity** (sets, factors) and **systems** (sets, number systems). Related concepts from MYP mathematics that could be used within the **numerical and abstract reasoning** branch include **approximation**, **equivalence**, **generalization**, **quantity**, **simplification**, **systems** and **validity**.

Topics and skills

The topics and skills in numerical and abstract reasoning will help students develop an understanding of:

- number sense and operations
- commutativity, associativity and distribution of operations
- estimation and approximation
- measurement and units
- decimal places and significant figures
- equivalence in number and representation
- proportional reasoning using ratios and percentages
- how proportional relationships lead to linear equations and modelling
- how ratios associate quantities that vary together
- solving versus satisfying an equation
- equivalent expressions
- application and limitations of accuracy
- money and financial literacy
- appreciation of errors.

MYP 1–3	MYP 4–5 (standard)	MYP 4–5 (extended)
	Number	
• Forms of numbers (fractions, decimals and percentages) and transforming between them	• Absolute values • Representing and solving inequalities, including	• Laws of exponents, including fractional/rational exponents

MYP 1–3	MYP 4–5 (standard)	MYP 4–5 (extended)
• Factors of numbers • Integers • Number operations • Prime numbers and prime factors • Greatest/highest common factor, lowest common multiple • Recurring decimals • Number lines and simple inequalities • Ratios • Exponents and powers • Squares and square roots • Time zones, clocks and timetables	compound and double inequalities • Irrational numbers • Surds, roots and radicals, including simplifying • Standard form (scientific notation) • Laws of exponents, including integer and negative exponents • Number systems notation • Direct and inverse proportion • Number sequences (prediction, description)	• Logarithms, including laws of logarithms and the use of technology to find values • Upper and lower bounds
Algebra		
• Operating with algebraic expressions • Forming equations • Transposing and solving simple equations • Substitution into expressions • Expanding brackets • Factorizing algebraic expressions • Using formulae • Flowcharts and simple algorithms	• Factorizing quadratic expressions • Solving quadratic equations • Changing the subject of an equation	• Arithmetic and geometric sequences
Enrichment		
Number • Exponential equations • Rationalizing the denominator • Number bases Algebra • Arithmetic and geometric series and summation • Sigma notation • Convergence and divergence		

Thinking with models

Models are depictions of real-life events using expressions, equations or graphs while a function is defined as a relation or expression involving one or more variables. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and tables represents different ways to communicate mathematical ideas. In order to use and develop mathematical models, students will require knowledge of concepts, topics and skills in algebra. Teachers should assist students' understanding of models by using real-life contexts for the application of algebraic knowledge and skills in problem-solving situations.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the **thinking with models** branch include **aesthetics** (patterns and sequences, graphs), **change** (algebraic expressions, transformations), **connections** (patterns and sequences, functions and graphs), **systems** (functions, series), and **time, place, and space** (functions, equations). Related concepts from MYP mathematics that could be used within the **thinking with models** branch include **change**, **equivalence**, **patterns**, **quantity**, **representation**, **simplification**, and **systems**.

Topics and skills

The topics and skills in thinking with models will help students develop an understanding of:

- representation and shape of functions
- algorithmic thinking
- transformations of functions
- modelling with functions
- applications and limitations of models.

<table>
<tr><th>MYP 1–3</th><th>MYP 4–5 (standard)</th><th>MYP 4–5 (extended)</th></tr>
<tr><td rowspan="3">Not considered age-appropriate</td><td>• Mappings
• Function notation
• Linear functions
• $y = mx + c$ (see also spatial reasoning)
• Parallel and perpendicular lines (see also spatial reasoning)
• Systems of equations/ simultaneous equations
• Quadratic functions
• Algorithms</td><td>• Representation and shape of more complex functions
• Transformation of quadratic functions
• Rational functions
• Graphing trigonometric functions
• Linear programming, including inequalities
• Networks—edges and arcs, nodes/ vertices, paths
• Calculating network pathways
• Weighted networks
• Domain and range</td></tr>
<tr><td colspan="2">Enrichment</td></tr>
<tr><td colspan="2">• Exponential functions
• Transforming cubic and trigonometric functions
• Functions including In and e
• Composite functions
• Inverse functions
• Networks and probability, Pascal's triangle
• Logarithmic functions</td></tr>
</table>

Spatial reasoning

Spatial reasoning skills provide students with the tools for analysing, measuring and transforming geometric quantities in two and three dimensions.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within the **spatial reasoning** branch include **aesthetics** (geometric shapes, transformations), **change** (identities, transformations), **communities** (angle properties, triangle properties), **creativity** (transformations, similarity and congruency), **identity** (unit circle, identities), **perspective** (coordinate geometry, similarity and congruency), and **time, place and space** (three-dimensional coordinate geometry, transformations). Related concepts from MYP mathematics that could be used within the **spatial reasoning** branch include **change**, **equivalence**, **model**, **patterns**, **quantity**, **representation**, **space** and **systems**.

Topics and skills

The topics and skills in spatial reasoning will help students develop an understanding of:

- construction and manual skills
- visualisation and representation of 3D shapes
- moving between dimensions
- algorithmic thinking in trigonometry.

MYP 1–3	MYP 4–5 (standard)	MYP 4–5 (extended)
	Geometry	
• Classifying shapes and angles • Calculations with angle properties • Parallel lines and transversals • Perimeter (circumference), area and volume • Surface area and nets • Coordinates • Symmetry and reflection	• Metric conversions • Volume of regular polyhedra • Similarity and congruence • Coordinate geometry, including distance, midpoint and gradient formulae • Movement on a plane—isometric transformations, enlargements and tessellations • $y = mx + c$, gradients and intercepts (see also functions and models) • Gradient of parallel lines • Circle geometry • Rotation around a given point	• Volume and capacity (additional shapes) • Enlargement around a given point • Enlargement by a rational factor • Gradients of perpendicular lines • Identical representation of transformations
	Trigonometry	
Not considered age-appropriate	• Triangle properties • Bearings • Pythagoras' theorem • Trigonometric ratios in right-angled triangles	• Converse of Pythagoras' theorem • Sine rule and cosine rule, including applications (link to trigonometric functions)
	Enrichment	
Geometry • Fractals (informal introduction)		

MYP 1–3	MYP 4–5 (standard)	MYP 4–5 (extended)
		• Dilation • Inscribing and circumscribing shapes • Arc length and sector using radians • Polar coordinates • Vector notation • Vector spaces Trigonometry • Three-dimensional coordinate geometry • Area of a triangle rule • Unit circle • Radians • Equation of a circle with centre at the origin • Trigonometric identities

Reasoning with data

This branch of mathematics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events.

Through the study of statistics, students should develop skills associated with the collection, organization and analysis of data, enabling them to present information clearly and to discover patterns. Students will also develop critical-thinking skills, enabling them to differentiate between what happens in theory (probability) and what is observed (statistics).

Students should understand both the power and limitations of statistics, becoming aware of their legitimate use in supporting and questioning hypotheses, but also recognizing how statistics can be used to mislead as well as to counter opinions and propaganda.

Students should use these skills in their investigations and are encouraged to use ICT whenever appropriate.

Links to MYP concepts

Key concepts from other MYP subjects that could be used within **reasoning with data** branch include **communication** (representation, probability of events), **communities** (samples, populations), **connections** (probability of successive trials, measures of central tendency), **development** (probability of successive trials, population sampling), **global interaction** (population sampling, representations) and **systems** (probability of events, conditional probability). Related concepts from MYP mathematics that could be used within the **reasoning with data** branch include **approximation**, **change**, **equivalence**, **generalization**, **model**, **patterns**, **quantity**, **representation**, **simplification**, **systems** and **validity**.

Topics and skills

The topics and skills in reasoning with data will help students develop an understanding of:

- variability and randomness
- causation versus correlation
- inferences and informal inferential reasoning
- prediction and hypothesis testing
- sampling, resampling and aggregation
- the role of context in statistical inquiry

- the connection between purpose and utility
- outliers and how to separate the signal from the noise
- critical literacy in statistics, considering sources and evaluating techniques
- data distribution and how they can be analysed or compared
- the law of large numbers
- theoretical and experimental probabilities
- listing strategies.

MYP 1–3	MYP 4–5 (standard)	MYP 4–5 (extended)
• Simple discrete data and classifications • Data collection and generation (including surveys) • Graphical representations (including: pie charts, bar charts, stem and leaf plots, pictograms) • Data visualizations and infographics • Data processing: measure of central tendency (mean, mode and median) for discrete and grouped data • Measures of dispersion: range • Limitations and context in statistical enquiry • Qualitative handling of probability • Probability of simple events • Sample spaces • Probability scale, including significance of number • Theoretical probability and experimental probability	• Sampling techniques • Data manipulation and misinterpretation • Graphical representations (including: bivariate graphs, scatter graphs, box plots, cumulative frequency graphs) • Lines of best fit • Data processing: quartiles and percentiles • Measures of dispersion: interquartile range • Correlation, qualitative handling • Relative frequency • Response rates • Sets, including notation and operations up to three sets • Probability with Venn diagrams, tree diagrams and sample spaces • Mutually exclusive events • Combined events	• Measure of dispersion: standard deviation • Correlation, quantitative handling, using technology • Histograms for continuous fixed interval groups • Addition and multiplication rule—conditional probability • Probability calculations • Dependent and independent events
Enrichment		
• Manual calculation of standard deviation, correlation coefficient • Histograms and frequency polygons • Covariance • Interpolation and extrapolation • Confirmation bias • Counting principles • Combinatorics • Permutations • Factorials • Geometric mean		

MYP 1–3	MYP 4–5 (standard)	MYP 4–5 (extended)
• Regression—interpretation of results		

Subject-specific guidance

Effective use of information and communication technology in mathematics

The appropriate use of computers, computer applications and calculators can improve the understanding of all students. Depending upon the school resources, ICT should be used whenever appropriate:

- as a means of expanding students' knowledge of the world in which they live
- as a channel for developing concepts and skills
- as a powerful communication tool.

ICT provides a wide range of resources and applications for teachers to explore in order to enhance teaching and learning. In mathematics, ICT can be used as a tool to perform complicated calculations, solve problems, draw graphs, and interpret and analyse data. ICT can also be helpful to:

- investigate data and mathematical concepts
- obtain rapid feedback when testing out solutions
- observe patterns and make generalizations
- move between analytical and graphical representations
- visualize geometrical transformations.

In addition, the appropriate use of ICT can enhance students' communication skills, assisting them in the collection, organization and analysis of information and in the presentation of their findings.

However, for ICT to be a useful tool for learning, students need to be familiar with the resources and applications, and know how and when to use them. Students should be able to decide when the use of ICT is appropriate and when alternative methods such as pencil and paper, mental calculation or diagrams should be used. Therefore, it is important that teachers show students how to use these resources effectively while supporting the development of their intellectual skills.

ICT can support students with learning support requirements who have difficulties understanding a particular concept or who would benefit from further practice. It can also provide the extra challenge for gifted and talented students to explore further ideas and concepts. "Adaptive technologies" can enable students with severe learning disabilities to become active learners in the classroom alongside their peers. For more information about adaptive technologies and learning support requirements, please refer to the Inclusive education page on the programme resource centre.

Depending on the school facilities and the availability of ICT resources, teachers are encouraged to use ICT whenever possible and appropriate as a means of enhancing learning.

Some of the possible ICT resources in mathematics might include:

- databases and spreadsheets
- graph-plotter software
- dynamic geometry software
- mathematics content-specific software
- graphic display calculators (GDCs)
- internet search engines
- CD-ROMs
- word processing or desktop publishing

- graphic organizers
- computer algebra system (CAS).

Assessment tasks in MYP mathematics

Generally, criteria A, B and D are assessed with different kinds of tasks. Criterion C is often used to assess constructed responses and reports in combination with criterion B or D.

Criterion	Typical assessment tasks	Notes
Criterion A: Knowing and understanding	Classroom tests Examinations Assignments that include both familiar and unfamiliar situations	Teachers who choose to use criterion A with criterion B should be able to clearly define which aspects of the task will be assessed with each criterion, ensuring that the task is rigorous enough to allow students to achieve the highest levels of both criteria.
Criterion B: Investigating patterns	Mathematical investigations of some complexity that allow students: • to choose their own mathematical techniques • to reason from the specific to the general.	Assessment tasks could have a variety of solutions and should enhance independent mathematical thinking.
Criterion C: Communicating	Investigations and real-life problems Reports that: • require logical structure • allow multiple forms of representation to present information.	Criterion C is often used when students present a report, for example, that requires a logical structure in order to be followed and that would allow for several forms of representation to be used to present information.
Criterion D: Applying mathematics in real-life contexts	Opportunities to use mathematical concepts to solve real-life problems	For example, modelling or curve-fitting tasks based in authentic contexts. Mathematics can be used to model many situations (for example, painting a room, analysing mobile telephone tariff plans, triangulation, diet plans).

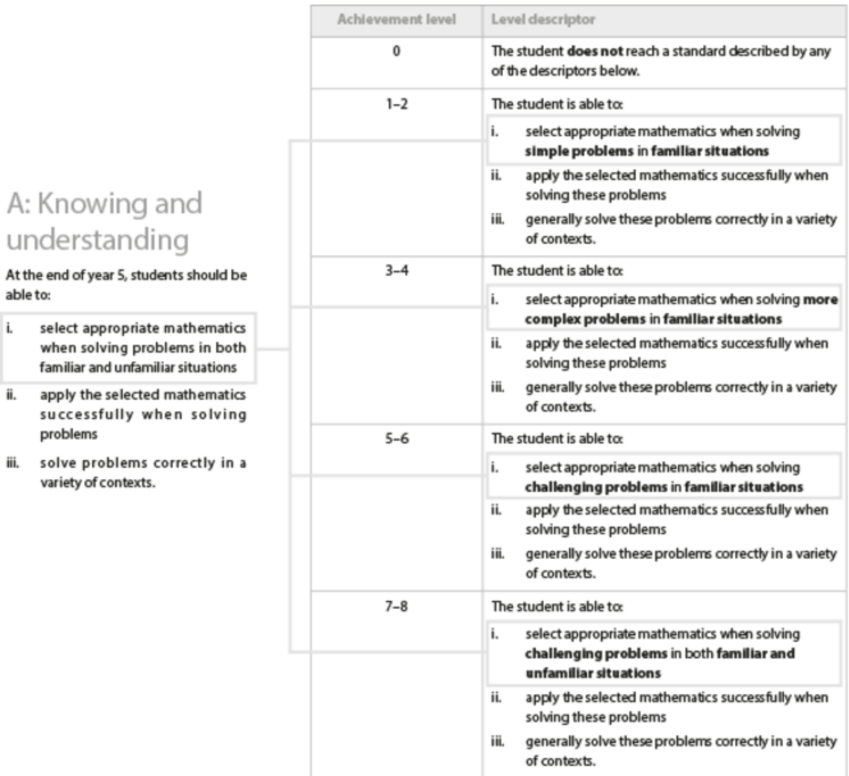
For further information please refer to the "Assessed curriculum" part of this guide, and to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Alignment of objectives and assessment criteria

In the MYP, assessment is closely aligned with the written and taught curriculum. Each strand from MYP mathematics has a corresponding strand in the assessment criteria for this subject group. Figure 3 illustrates this alignment and the increasingly complex demands for student performance at higher achievement levels.

Figure 3

Mathematics objectives and criteria alignment



Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. select appropriate mathematics when solving **simple problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
3–4	The student is able to: i. select appropriate mathematics when solving **more complex problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
5–6	The student is able to: i. select appropriate mathematics when solving **challenging problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
7–8	The student is able to: i. select appropriate mathematics when solving **challenging problems** in both **familiar and unfamiliar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material** and to *Further guidance for developing MYP assessed curriculum* (April 2015, updated September 2016), which can be found under **MYP resources**>**Learning and teaching**>**General material**>**Further guidance**.

Assessment criteria overview

Assessment for mathematics courses in all years of the programme is criterion-related, based on four equally weighted assessment criteria.

Criterion A	Knowing and understanding	**Maximum 8**
Criterion B	Investigating patterns	**Maximum 8**
Criterion C	Communicating	**Maximum 8**
Criterion D	Applying mathematics in real-life contexts	**Maximum 8**

Subject groups **must** assess **all** strands of **all** four assessment criteria **at least twice** in **each year** of the MYP.

In the MYP, subject-group objectives correspond to assessment criteria. Each criterion has eight possible achievement levels (1–8), divided into four bands that generally represent limited (1–2); adequate (3–4); substantial (5–6); and excellent (7–8) performance. Each band has its own unique descriptor that teachers use to make "best-fit" judgments about students' progress and achievement.

This guide provides the **required assessment criteria** for years 1, 3 and 5 of MYP mathematics. In response to national or local requirements, schools may add criteria and use additional models of assessment. Schools must use the appropriate assessment criteria, as published in this guide, to report students' final achievement in the programme.

Teachers clarify the expectations for each summative assessment task with direct reference to these assessment criteria. Task-specific clarifications should clearly explain what students are expected to know and do. They might be in the form of:

- a task-specific version of the required assessment criteria
- a face-to-face or virtual classroom discussion
- a detailed task sheet or assignment.

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Mathematics assessment criteria: Year 1

Criterion A: Knowing and understanding

Maximum: 8

At the end of year 1, students should be able to:

i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. apply the selected mathematics successfully when solving problems

iii. solve problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. select appropriate mathematics when solving **simple problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
3–4	The student is able to: i. select appropriate mathematics when solving **more complex problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
5–6	The student is able to: i. select appropriate mathematics when solving **challenging problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
7–8	The student is able to: i. select appropriate mathematics when solving **challenging problems** in both **familiar and unfamiliar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.

Criterion B: Investigating patterns

Maximum: 8

At the end of year 1, students should be able to:

i. apply mathematical problem-solving techniques to recognize patterns

ii. describe patterns as relationships or general rules consistent with correct findings

iii. verify whether the pattern works for other examples.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **apply**, **with teacher support**, mathematical problem-solving techniques to recognize **simple patterns** ii. **state** predictions consistent with simple patterns.
3–4	The student is able to: i. **apply** mathematical problem-solving techniques to recognize **patterns** ii. **suggest** how these patterns work.
5–6	The student is able to: i. **apply** mathematical problem-solving techniques to recognize **patterns** ii. **suggest relationships** or general rules consistent with findings iii. verify whether patterns work for **another example**.
7–8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to recognize **correct patterns** ii. **describe patterns as relationships** or general rules consistent with **correct findings** iii. verify whether patterns work for **other examples**.

Note: A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 6 (for years 1 and 2).

Criterion C: Communicating

Maximum: 8

At the end of year 1, students should be able to:

i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written statements

ii. use appropriate forms of mathematical representation to present information

iii. communicate coherent mathematical lines of reasoning

iv. organize information using a logical structure.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. use **limited** mathematical language ii. use **limited forms** of mathematical representation to present information iii. communicate through lines of reasoning that are **difficult to understand**
3–4	The student is able to: i. use **some appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to present information **adequately** iii. communicate through lines of reasoning that are **able to be understood**, although these are **not always coherent**

Achievement level	Level descriptor
	iv. **adequately organize** information using a logical structure.
5–6	The student is able to: i. **usually** use **appropriate** mathematical language ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly** iii. communicate through lines of reasoning that are **usually coherent** iv. **present** work that is **usually organized** using a logical structure.
7–8	The student is able to: i. **consistently** use **appropriate** mathematical language ii. **consistently** use **appropriate forms** of mathematical representation to present information **correctly** iii. communicate **clearly** through **coherent** lines of reasoning iv. present work that is **consistently organized** using a logical structure.

Criterion D: Applying mathematics in real-life contexts

Maximum: 8

At the end of year 1, students should be able to:

i. identify relevant elements of authentic real-life situations
ii. select appropriate mathematical strategies when solving authentic real-life situations
iii. apply the selected mathematical strategies successfully to reach a solution
iv. explain the degree of accuracy of a solution
v. describe whether a solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. identify **some** of the elements of the authentic real-life situation ii. apply mathematical strategies to **find a solution** to the authentic real-life situation, **with limited success**.
3–4	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. apply mathematical strategies to **reach a solution** to the authentic real-life situation iii. **state**, **but not always correctly**, whether the solution makes sense in the context of the authentic real-life situation.
5–6	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **adequate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a valid solution** to the authentic real-life situation

Achievement level	Level descriptor
	iv. **describe** the degree of accuracy of the solution v. **state correctly** whether the solution makes sense in the context of the authentic real-life situation.
7–8	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **adequate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a correct solution** to the authentic real-life situation iv. **explain** the degree of accuracy of the solution v. **describe correctly** whether the solution makes sense in the context of the authentic real-life situation.

Mathematics assessment criteria: Year 3

Criterion A: Knowing and understanding

Maximum: 8

At the end of year 3, students should be able to:

i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. apply the selected mathematics successfully when solving problems

iii. solve problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. select appropriate mathematics when solving **simple problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
3–4	The student is able to: i. select appropriate mathematics when solving **more complex problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
5–6	The student is able to: i. select appropriate mathematics when solving **challenging problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
7–8	The student is able to: i. select appropriate mathematics when solving **challenging problems** in both **familiar and unfamiliar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.

Criterion B: Investigating patterns

Maximum: 8

At the end of year 3, students should be able to:

i. select and apply mathematical problem-solving techniques to discover complex patterns

ii. describe patterns as relationships and/or general rules consistent with findings

iii. verify and justify relationships and/or general rules.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **apply**, **with teacher support**, mathematical problem-solving techniques to discover **simple patterns** ii. **state predictions** consistent with patterns.
3–4	The student is able to: i. **apply** mathematical problem-solving techniques to discover **simple patterns** ii. **suggest relationships** and/or general rules consistent with **findings**.
5–6	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as relationships and/or general rules consistent with **findings** iii. **verify** these relationships and/or general rules.
7–8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as relationships and/or general rules consistent with **correct findings** iii. **verify** and **justify** these relationships and/or general rules.

Note: A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 4 (year 3 and higher). However, teachers should give enough direction to ensure that all students can begin the investigation.

For year 3 and higher, a student who describes a general rule consistent with incorrect findings will be able to achieve a maximum achievement level of 6, provided that the rule is of an equivalent level of complexity.

Criterion C: Communicating

Maximum: 8

At the end of year 3, students should be able to:

i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations

ii. use appropriate forms of mathematical representation to present information

iii. move between different forms of mathematical representation

iv. communicate complete and coherent mathematical lines of reasoning

v. organize information using a logical structure.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. use **limited** mathematical language ii. use **limited forms** of mathematical representation to present information

Achievement level	Level descriptor
	iii. communicate through lines of reasoning that are **difficult to interpret**.
3–4	The student is able to: i. use **some appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to present information **adequately** iii. communicate through lines of reasoning that are **able to be understood**, although these are **not always clear** iv. **adequately organize** information using a logical structure.
5–6	The student is able to: i. **usually** use **appropriate** mathematical language ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly** iii. move between different forms of mathematical representation **with some success** iv. communicate through lines of reasoning that are clear although **not always coherent or complete** v. present work that is **usually organized** using a logical structure.
7–8	The student is able to: i. **consistently** use **appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to **consistently** present information **correctly** iii. move **effectively** between different forms of mathematical representation iv. communicate through lines of reasoning that are **complete and coherent** v. present work that is **consistently organized** using a logical structure.

Criterion D: Applying mathematics in real-life contexts

Maximum: 8

At the end of year 3, students should be able to:

i. identify relevant elements of authentic real-life situations

ii. select appropriate mathematical strategies when solving authentic real-life situations

iii. apply the selected mathematical strategies successfully to reach a solution

iv. explain the degree of accuracy of a solution

v. explain whether a solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. identify **some** of the elements of the authentic real-life situation ii. apply mathematical strategies to **find a solution** to the authentic real-life situation, **with limited success**.
3–4	The student is able to:

Achievement level	Level descriptor
	i. identify the **relevant** elements of the authentic real-life situation ii. select, **with some success**, **adequate** mathematical strategies to model the authentic real-life situation iii. apply mathematical strategies to **reach a solution** to the authentic real-life situation iv. **describe** whether the solution makes sense in the context of the authentic real-life situation.
5–6	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **adequate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a valid solution** to the authentic real-life situation iv. **describe** the degree of accuracy of the solution v. **discuss** whether the solution makes sense in the context of the authentic real-life situation.
7–8	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **appropriate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a correct solution** iv. **explain** the degree of accuracy of the solution v. **explain** whether the solution makes sense in the context of the authentic real-life situation.

Mathematics assessment criteria: Year 5

Criterion A: Knowing and understanding

Maximum: 8

At the end of year 5, students should be able to:

i. select appropriate mathematics when solving problems in both familiar and unfamiliar situations

ii. apply the selected mathematics successfully when solving problems

iii. solve problems correctly in a variety of contexts.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. select appropriate mathematics when solving **simple problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
3–4	The student is able to: i. select appropriate mathematics when solving **more complex problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
5–6	The student is able to: i. select appropriate mathematics when solving **challenging problems** in **familiar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.
7–8	The student is able to: i. select appropriate mathematics when solving **challenging problems** in both **familiar and unfamiliar situations** ii. apply the selected mathematics successfully when solving these problems iii. generally solve these problems correctly in a variety of contexts.

Criterion B: Investigating patterns

Maximum: 8

At the end of year 5, students should be able to:

i. select and apply mathematical problem-solving techniques to discover complex patterns

ii. describe patterns as general rules consistent with findings

iii. prove, or verify and justify, general rules.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. **apply**, **with teacher support**, mathematical problem-solving techniques to discover **simple patterns** ii. **state predictions** consistent with patterns.
3–4	The student is able to: i. **apply** mathematical problem-solving techniques to discover **simple patterns** ii. **suggest general rules** consistent with **findings**.
5–6	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as general rules consistent with **findings** iii. **verify** the validity of these general rules.
7–8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as general rules consistent with **correct findings** iii. **prove**, or **verify** and **justify**, these general rules.

Note: A task that does not allow students to select a problem-solving technique is too guided and should result in students earning a maximum achievement level of 4 in year 5. However, teachers should give enough direction to ensure that all students can begin the investigation.

For year 5, a student who describes a general rule consistent with incorrect findings will be able to achieve a maximum achievement level of 6, provided that the rule is of an equivalent level of complexity.

Criterion C: Communicating

Maximum: 8

At the end of year 5, students should be able to:

i. use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations

ii. use appropriate forms of mathematical representation to present information

iii. move between different forms of mathematical representation

iv. communicate complete, coherent and concise mathematical lines of reasoning

v. organize information using a logical structure.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. use **limited** mathematical language ii. use **limited forms** of mathematical representation to present information iii. communicate through lines of reasoning that are **difficult to interpret**.
3–4	The student is able to:

Achievement level	Level descriptor
	i. use **some appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to present information **adequately** iii. communicate through lines of reasoning that are **complete** iv. **adequately organize** information using a logical structure.
5–6	The student is able to: i. **usually** use **appropriate** mathematical language ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly** iii. **usually** move between different forms of mathematical representation iv. communicate through lines of reasoning that are **complete and coherent** v. **present** work that is **usually organized** using a logical structure.
7–8	The student is able to: i. **consistently** use **appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to **consistently** present information **correctly** iii. move **effectively** between different forms of mathematical representation iv. communicate through lines of reasoning that are **complete, coherent and concise** v. **present** work that is **consistently organized** using a logical structure.

Criterion D: Applying mathematics in real-life contexts

Maximum: 8

At the end of year 5, students should be able to:

i. identify relevant elements of authentic real-life situations

ii. select appropriate mathematical strategies when solving authentic real-life situations

iii. apply the selected mathematical strategies successfully to reach a solution

iv. justify the degree of accuracy of a solution

v. justify whether a solution makes sense in the context of the authentic real-life situation.

Achievement level	Level descriptor
0	The student **does not** reach a standard described by any of the descriptors below.
1–2	The student is able to: i. identify **some** of the elements of the authentic real-life situation ii. apply mathematical strategies to **find a solution** to the authentic real-life situation, **with limited success**.
3–4	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select, **with some success**, **adequate** mathematical strategies to model the authentic real-life situation

Achievement level	Level descriptor
	iii. apply mathematical strategies to **reach a solution** to the authentic real-life situation iv. **discuss** whether the solution makes sense in the context of the authentic real-life situation.
5–6	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **adequate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a valid solution** to the authentic real-life situation iv. **explain** the degree of accuracy of the solution v. **explain** whether the solution makes sense in the context of the authentic real-life situation.
7–8	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **appropriate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a correct solution** to the authentic real-life situation iv. **justify** the degree of accuracy of the solution v. **justify** whether the solution makes sense in the context of the authentic real-life situation.

MYP eAssessment

Students seeking **IB MYP course results** for MYP mathematics courses complete an on-screen examination in which they can demonstrate their achievement of subject-group objectives. Successful results can contribute to students' attainment of the **IB MYP certificate**.

For mathematics, optional on-screen examinations are written with the expectation that students have completed the mathematics framework for standard or extended mathematics. This verification of learning assures accurate and consistently applied standards.

Mathematics examination blueprint

MYP on-screen examinations are constructed as a series of tasks that sample, simulate or replicate internal assessment practices. The assessments follow an agreed structure that provides a clear framework for developing each examination. The distribution of marks within each eAssessment may vary by no more than three marks from those displayed in the blueprint.

As part of an ethical assessment model, these assessment blueprints ensure consistency and transparency, and they guarantee a balanced approach in measuring students' achievement with respect to MYP objectives. MYP on-screen examination blueprints document the close connection of large-scale assessment with subject-group objectives, classroom learning engagements and the programme's rigorous internal assessment requirements.

These blueprints enable teachers and students to review the nature and purpose of MYP eAssessment. They provide an important resource for helping students to prepare for on-screen examinations, focusing attention on subject-group criteria and assessment strategies in each subject group.

Overview

The following table illustrates how on-screen examinations in mathematics assessment are structured.

Task	Marks	Main criteria assessed	Criterion marks
Knowing and understanding	31–35*	A	25
		C	6–10*
Applying mathematics in real-life contexts	31–35*	D	25
		C	6–10*
Investigating patterns	31–35	B	25
		C	6–10
	100		

*Note that criterion C is assessed equally across all tasks to mark a total of 25 marks.

Examination sources, tools and tasks

Sources

A variety of sources will feature in each assessment and could include the following.

- Animations
- Data tables

- Graphs
- Simulations
- Static images
- Videos

Tools

A variety of response tools will be available to students including but not limited to an on-screen calculator, a measuring tool, drawing canvasses, a mathematics canvas, a graph plotter and a table drawing tool.

Tasks

Knowing and understanding

The first task assesses students' knowledge and understanding of mathematics, but marks may be awarded against the other criteria when appropriate to the skills used in answering a question. For example, a question assessing knowledge and understanding may also require students to move between different forms of mathematical representation.

Applying mathematics in real-life contexts

The second task assesses students' ability to apply mathematics in a real-life context, which is typically connected to the global context for the session. Students may be required to produce pieces of extended writing to evaluate and justify the validity of mathematics models.

Investigating patterns

Investigative skills in mathematics will be assessed in the final task. The abstract questions in this task will contain a greater degree of scaffolding than would be appropriate in the classroom to allow students of different abilities to access the task.

Conventions for on-screen examinations in MYP mathematics

Standardized symbols, notation and terminology for mathematics

MYP eAssessments use the standards adopted by the IB from a system of notation based on ISO 80000 (International Organization for Standardization, 2009). Students are expected to recognize this notation in mathematics, and teachers should introduce this notation as a regular part of MYP courses in these subject groups as appropriate.

For on-screen examinations, symbols, units and equations—where appropriate—are provided on a toolbar to ensure consistent usage and authentic age-appropriate mathematical communication. If an examination question requires additional symbols or notations, they will be defined and explained within the context of the relevant task.

Candidates must always use correct mathematical notation, not calculator notation. Candidates should be familiar with scientific notation, also referred to as standard form as follows.

$a \times 10^k$ where $1 \leq a \leq 10$ and $k \in \mathbb{Z}$

Answers will require an appropriate use of significant figures or decimal places based on the demands of the question. Unless otherwise indicated, final answers are to be given correct to three significant figures. Estimation is to be completed by rounding; truncation will not be rewarded.

Correct use of subscript and superscript is expected in all relevant mathematical contexts.

Where specific currency symbols are required, they will be provided as a button on the toolbar.

The following list does not constitute additional curriculum specifications beyond the MYP mathematics framework published in this guide. Rather, the symbols below depict the universe of mathematical symbols that could be used in relevant questions and the symbols that will be available for students to use in their responses. They provide a common shared communication convention for MYP eAssessment.

Table 7

Standardized symbols

Symbol	Meaning
$\mathbb{N}$	The set of positive integers and zero, $\{0, \quad 1, \quad 2, \quad 3 \quad ...\}$
$\mathbb{Z}$	The set of integers, $\{0, \quad \pm 1, \quad \pm 2, \quad \pm 3 \quad ...\}$
$\mathbb{Q}$	The set of rational numbers
$\mathbb{R}$	The set of real numbers
$+$	Plus
$-$	Minus
$\pm$	Plus or minus
$a \times b$	a multiplied by/times b Note: also accepted $a \cdot b$ (half-high dot) ab Not accepted: a b
a b a/b	Divided by Note: also accepted $a \times b^{-1}$
$=$	Is equal to
$\neq$	Is not equal to
$\equiv$	Is identical to
$\approx$	Is approximately equal to
~	Is proportional to
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to
∞	Infinity
$0.\dot{8}\dot{1}$	Recurring decimal, where the dot appears over the first and last repeating numeral
$\sphericalangle ABC$	Angle at vertex B in the triangle ABC Note: the angle is not oriented, it holds that $\sphericalangle ABC = \sphericalangle CBA$
$\bar{AB}$	Line segment from A to B
$\overrightarrow{AB}$	Vector from A to B
$\mathbf{a}$	Vector a Note: An arrow above the letter symbol can be used instead of bold face type to indicate a vector $(\vec{a})$
$\mathbf{a} \cdot \mathbf{b}$	Dot product of a and b

Symbol	Meaning
	Note: Must be in bold to distinguish from simple multiplication
Σ	Sigma, sum of
$\sum_{i}^{n} = {}_{1}a_{i}$	$a_1 + a_2 + \ldots + a_n$ sum of $a_1, a_2, \ldots, a_n$
a^p	a to the power of p Note: use of calculator terminology ∧ will not be accepted
$\sqrt{a}$	Square root Note: $a^{1/2}$ will also be accepted
$\sqrt[n]{a}$	nth root of a Note: If the symbol acts on a composite expression, parentheses or brackets must be used to avoid ambiguity Also accepted $a^{1/n}$
$\bar{X}$	Mean value of x
σ	Standard deviation
$\|a\|$	Absolute value of a Also: vector magnitude
$f,\quad g,\quad h\quad \approx$	Functions, models, for example, $f(x) =$ Note: $y =$ also accepted where not dictated by stimulus or question
$f^{-1},\quad g^{-1}$	Inverse functions
$g \circ f$	Composite function Note: $(g \circ f)(x) = g(f(x))$
e	Base of natural logarithm
$\log_2(x)$	Logarithm to the base 2 of argument x
$\ln(x)$	Natural logarithm of x
π	Pi, ratio of the circumference of a circle to its diameter
$\sin(x)$ $\cos(x)$ $\tan(x)$ $\sin^{-1}(x)$ $\cos^{-1}(x)$ $\tan^{-1}(x)$	Sine of x, cosine of x, tangent of x Inverse functions of above Note: arcsin, and so on, will be accepted but not provided on the calculator or toolbar
∈	Is an element of
∉	Is not an element of
∅	The empty (null) set
$n(\mathrm{A})$	The number of elements in the finite set A
𝕌	The universal set
∪	Union
∩	Intersection

Symbol	Meaning
$\subset$	Is a proper subset of
$\subseteq$	Is a subset of
A'	The complement of the set A

Mathematics subject-specific grade descriptors

Subject-specific grade descriptors serve as an important reference in the assessment process. Through careful analysis of subject-group criteria and the general grade descriptors, they have been written to capture and describe in a single descriptor the performance of students at each grade for each MYP subject group.

For on-screen examination subjects, teachers are required to submit predicted grades. When considering predicted grades, teachers should consider their own assessment of students during MYP year 4 and the first part of MYP year 5 and, allowing for subsequent academic development, are asked to predict the outcome of eAssessment for their students with reference to the subject-specific grade descriptors. This prediction helps the IB to check the alignment between teachers' expectations and the IB's assessment outcome and, as such, forms an essential strategy for ensuring reliable results.

Subject-specific grade descriptors are also the main reference used to select grade boundaries for each discipline in each assessment session. During this process, the grade award team compares student performance against descriptors of achievement at grades 2 and 3; 3 and 4; and 6 and 7 (other boundaries are set at equal intervals between these key transitions). The grade award process is able to compensate for variations in challenge between examinations and in standards applied to marking (both between subjects and for a particular subject across sessions) by setting boundaries for each discipline and examination session, with reference to real student work.

Subject-specific grade descriptors tie eAssessment to criterion-related assessment and to MYP assessment criteria and level descriptors, which put the programme's criterion-related assessment philosophy into practice.

Grade	Descriptor
1	Produces work of a very limited quality. Conveys many significant misunderstandings or lacks understanding of most concepts and contexts. Very rarely demonstrates evidence of mathematical thinking. Very inflexible, rarely shows evidence of knowledge or skills.
2	Produces mathematical work of limited quality. Communicates limited understanding of some concepts and contexts. Demonstrates limited evidence of mathematical thinking. Limited evidence of transfer of mathematical knowledge and application of skills.
3	Produces mathematical work of an acceptable quality. Communicates basic understanding of many concepts and contexts with occasional evidence of appropriate application of mathematical techniques and terminology, with occasional significant misunderstandings or gaps. Begins to demonstrate some analytical thinking when problem-solving and investigating. Begins to transfer mathematical knowledge and apply skills, requiring support even in familiar classroom situations.
4	Produces good-quality mathematical work. Communicates basic understanding of most concepts and contexts with evidence of appropriate application of mathematical techniques and terminology, with few misunderstandings and minor gaps. Often demonstrates analytical thinking when problem-solving and investigating. Transfers some mathematical knowledge and applies skills in familiar classroom situations, but requires support in unfamiliar situations.
5	Produces generally high-quality mathematical work. Communicates good understanding of concepts and contexts demonstrating proficient application of

Grade	Descriptor
	mathematical techniques and terminology. Demonstrates analytical thinking and logical processes, sometimes with sophistication, when problem-solving and investigating. Usually transfers mathematical knowledge and applies skills, with some independence, in familiar classroom and real-world situations.
6	Produces high-quality, occasionally insightful mathematical work. Communicates extensive understanding of concepts and contexts demonstrating proficient application of mathematical techniques and terminology. Demonstrates analytical thinking and logical processes, frequently with sophistication when problem-solving and investigating. Transfers mathematical knowledge and applies skills, often with independence, in a variety of familiar and unfamiliar classroom and real-world situations.
7	Produces high-quality work that frequently uses mathematics insightfully. Communicates comprehensive, nuanced understanding of concepts and contexts demonstrating proficient application of mathematical techniques and terminology. Consistently demonstrates sophisticated analytical thinking and logical processes when problem-solving and investigating. Frequently transfers mathematical knowledge and applies skills, with independence and expertise, in a variety of complex classroom and real-world situations.

Related concepts in mathematics

Related concept	Definition
Approximation	A quantity or a representation that is nearly but not exactly correct.
Change	A variation in size, amount or behaviour.
Equivalence	The state of being identically equal or interchangeable, applied to statements, quantities or expressions.
Generalization	A general statement made on the basis of specific examples.
Models	Depictions of real-life events using expressions, equations or graphs.
Patterns	Sets of numbers or objects that follow a specific order or rule.
Quantity	An amount or number.
Representation	The manner in which something is presented.
Simplification	The process of reducing to a less complicated form.
Space	The frame of geometrical dimensions describing an entity.
Systems	Groups of interrelated elements.
Validity	Using well-founded, logical mathematics to come to a true and accurate conclusion or a reasonable interpretation of results.

Mathematics glossary

Term	Definition
Authentic real life	Relevant, meaningful and grounded in reality.
Challenging	Demanding problems of high complexity that require students to have mathematical insight to be able to use knowledge and/or skills taught.
Communicate	Express oneself in such a way that one is readily and clearly understood. Convey information about the exchange of thoughts, messages or information through, for example, speech, signals, writing or behaviour.
Context	The setting of the problem.
Familiar situations	Problems similar to those seen previously in which students are required to use knowledge and/or skills they have been taught.
Form	The understanding that the underlying structure and shape of an entity is distinguished by its properties. Provides opportunities for students to appreciate the aesthetic nature of the constructs used in mathematics.
Forms of mathematical representation	Words, formulae, diagrams, tables, charts, graphs and models used to represent mathematical information.
Investigation	A task where, to varying degrees, students are given opportunities to pose questions, select problem-solving techniques, discover patterns, make generalizations and communicate their findings.
Justification	Valid reasons or evidence that support the conclusion and explain why the rule works.
Lines of reasoning	A connected sequence of steps.
Logic	The basic tool used in mathematics to make conclusions about numbers, shapes and variables. Structures the reasoning process through which knowledge is built and enables students to assess the truth of conclusions and transfer mathematical learning to other situations.
Logical structure	A general layout that prevents the need for going back and forth (between the task sheet and the student work and within the student work) in order to understand and follow the work.
Mathematical language	The use of notation, symbols, terminology and verbal explanations.
Model	Represent.
Pattern	The underlining order, regularity or predictability of the elements of a mathematical system.

Term	Definition
	The repetitive features of patterns can be identified and described as relationships or general rules.
Problem-solving techniques	Strategies students use to solve problems (for example, making a table or chart, solving a simpler problem, working backwards, drawing a picture, guessing and checking, and so on).
Proof	The use of a sequence of logical steps to obtain the required result in a formal way.
Relationships	Refers to the connections between quantities, properties or concepts; these connections may be expressed as models, rules or statements. Provides opportunities for students to explore patterns in the world around them.
Teacher support	Advice given by the teacher to aid students with elements of the task (for example, to allow a student to start solving the problem).
Test	Verify whether a rule works for a variety of values.
Unfamiliar situations	New contexts in which students are required to use knowledge and/or skills they have been taught.
Unit test	A test comprised of skills from only one branch of mathematics from the framework.
Valid	A plausible solution in the context of the situation.

MYP command terms for mathematics

Command term	Definition
Annotate	Add brief notes to a diagram or graph.
Apply	Use knowledge and understanding in response to a given situation or real circumstances. Use an idea, equation, principle, theory or law in relation to a given problem or issue. (See also “Use”.)
Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Construct	Display information in a diagrammatic or logical form.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Derive	Manipulate a mathematical relationship to give a new equation or relationship.
Describe	Give a detailed account or picture of a situation, event, pattern or process.
Discuss	Offer a considered and balanced review that includes a range of arguments, factors or hypotheses. Opinions or conclusions should be presented clearly and supported by appropriate evidence.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value for an unknown quantity.
Explain	Give a detailed account including reasons or causes. (See also “Justify”.)
Identify	Provide an answer from a number of possibilities. Recognize and state briefly a distinguishing fact or feature.
Justify	Give valid reasons or evidence to support an answer or conclusion. (See also “Explain”.)
Label	Add title, labels or brief explanation(s) to a diagram or graph.
Measure	Obtain a value for a quantity.
Organize	Put ideas and information into a proper or systematic order.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result of an upcoming action or event.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Select	Choose from a list or group.
Show	Give the steps in a calculation or derivation.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship and should include relevant features.

Command term	Definition
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Trace	Follow and record the action of an algorithm.
Use	Apply knowledge or rules to put theory into practice. (See also "Apply".)
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

On-screen examinations in mathematics will draw from the full list of MYP command terms available in *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

Selected reading

For further information please refer to *MYP: From principles into practice*, which can be found in the programme resource centre under **MYP resources**>**Learning and teaching**>**General material**.

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